

Ignorance is Not Bliss: An Analysis of Central-Place Foraging Algorithms

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Abstract—Central-place foraging (CPF) is a canonical task in collective robotics with applications to planetary exploration, automated mining, warehousing, and search and rescue operations. We compare the performance of three Central-Place Foraging Algorithms (CPFAs), variants of which have been shown to work well in real robots: spiral-based, rotating-spoke, and random-ballistic. To understand the difference in performance between these CPFAs, we define the *price of ignorance* and show how this metric explains our previously published empirical results. We obtain upper-bounds for expected complete collection times for each algorithm and evaluate their performance in simulation. We show that site-fidelity (i.e. returning to the location of the last found target) and avoiding search redundancy are key-factors that determine the efficiency of CPFAs. Our formal analysis suggests the following efficiency ranking from best to worst: spiral, spoke, and the stochastic ballistic algorithm.

I. INTRODUCTION

Autonomous central-place foraging (CPF) is a fundamental task in collective robotics that involves the discovery, collection, and transportation of targets to a collection zone [1]. Central-Place Foraging Algorithms (CPFAs) have recently received increased attention as resource collection on other planets, moons, and asteroids by robots is planned by space agencies to enable human exploration. Mining by autonomous vehicles and inventory collection in automated warehouses are essentially CPF tasks in that they require efficient collection and transportation of targets distributed within an area. Search and rescue, collection of bomb fragments for analysis, and robotic agriculture also motivate the study of distributed retrieval tasks. In biology, immune systems searching for pathogens and ant-colonies searching for targets can also be understood by analysing CPFAs.

Empirical work in real robots leads us to investigate three simple algorithms: the Distributed Archimedean Spiral Algorithm (SPIRALCPFA) [2], [3], Spoke Central Place Foraging Algorithm (SPOKECPFA) [4], and Random Ballistic Central Place Foraging Algorithm (RANDCPFA) [5]. Variants of these three algorithms performed well in the NASA Swarmathon, a swarm robot foraging competition that allowed us to make direct comparisons among many different foraging algorithms in simulations and in physical robots [4].

Our formal analysis allows us to predict the performance of these CPFAs for large areas and swarms of robots for which experiments are currently impractical. We formalise two principles observed in our empirical work: the importance of site-fidelity (returning to the location of the

last found target) and the adverse effects of oversampling (repeatedly searching the same area). To aid our analysis, we introduce the *price of ignorance* metric. This metric is the ratio of the performance of a given algorithm to that of an idealised omniscient algorithm[†]. This quantifies the penalty paid by each algorithm for not knowing target locations, which for the complete collection task is the major determinant of CPFA performance. Our preliminary results using this metric appeared in [8].

For each of the CPFAs, the proofs presented here provide relative upper-bounds on the expected performance of teams of robots. These upper-bounds suggest the ordering of algorithm performance in idealised scenarios. To test whether the ranking of upper-bounds holds, we run experiments using the Autonomous Robots Go Swarming (ARGoS) [9] simulator for each of the CPFAs. In combination, the asymptotic analysis and ARGoS simulations give us insight into how CPFAs perform in theory and in practice.

The technical details of our model along with our formal analysis is presented in Section II. We describe our empirical methods in Section III and present the results of our analysis in Section IV. Finally we discuss our findings in Section V.

Related Work. Seminal contributions in search and distributed foraging have emerged in Operations Research [10], Physics [11], Computational Geometry [12] and Robotics [13]–[15] (among other areas). Central place foraging has been of fundamental interest to researchers of Swarm Intelligence because of its deep connections to social insect behaviour [16]. Generating an optimal search path that maximises the probability of detecting a target in non-trivial environments within a fixed time-frame is NP-complete, and minimising the mean time to detection is NP-hard [17], [18]. Therefore, search and CPF use heuristics.

Rybski et al. [19], and Hecker and Moses [15] demonstrate that site-fidelity can dramatically reduce search times in robot CPF in simulation and hardware experiments. So far this effect has mostly been argued empirically [20], and our result provides some of the first theoretical evidence to explain this phenomenon.

Ghosh and Klein [21] provide a review of planar search algorithms, which is a critical component of CPFAs. Spiral

[†]We note that the price of ignorance is similar to the notion of *competitiveness* for online algorithms [6], which has been used in the context of foraging in [7]. However, in our application, rather than measuring competitiveness with respect to the amount of advice available (as in [7]) we measure competitiveness with respect to the knowledge of the locations of resources and in the ability of the foraging algorithm to avoid repeatedly searching the same locations in the foraging arena. Additionally, we assume a uniformly random pile placement, which is different from the adversarial setting often analysed for online algorithms.

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TABLE I: Notation.

χ	Price of ignorance	T_s	Total search time
T_t	Total transport time	T_{tot}	Total foraging time
N	Number of searchers	s	Robot speed
R	Radius of the circular arena	r	Detection radius of robot
f	Total number of targets	m	Total number of clusters
a	Area of a single target	p_s	Probability of site-fidelity

search has long been known to be an optimal search strategy for *individual* agents searching for *single* targets, both from the standpoint of computational geometry [22] and more recently as a practical algorithm for real robots [23]. The optimality of spiral search for multiple agents searching for a line in the plane, and a point on a line, has also been proved [12]. We have examined the generalisation to multiple robots empirically [2]. To our knowledge this is the first formal analysis of multi-agent spiral and spoke CPFAs.

Feinerman et al. [24], also argue in favour of deterministic algorithms using formal analysis and establish a lower bound of $\Omega(Rf + R^2f/N)$ for the time it takes to collectively forage adversarially placed targets. In a slightly relaxed but biologically motivated setting of uniformly random cluster placement around the collection zone, we show that SPIRALCPFA circumvents this bound and takes only $O(R^2/N + R\max\{f/N, 1\})$ time in expectation for complete collection. In fact, Aggarwal et al. recently showed that when the targets are clustered into a pile of diameter Δ , then even under adversarial placement, there exists a single-agent deterministic foraging strategy which can locate this pile in $\Theta(R^2/\Delta)$ time steps [25].

II. THEORETICAL ANALYSIS

A. Formal Model

We assume N holonomic robots, which are instructed to search within a predefined area of radius R . We assume a simple obstacle-free circular arena with a central, stationary collection-zone. Robots begin at the collection zone and transport the targets they discover to the collection zone. The location of the collection zone, the number N (along with a unique identifier per robot) and the radius of the arena is known to all robots.

We distribute f targets evenly among m clusters. These clusters are placed uniformly at random within the arena. Targets are detectable when robots are within sensor range, r , which defines an area a around each target. Targets are stationary and depleted on collection. The number of targets initially available for collection (and their locations) is fixed but unknown to the robots.

We assume that the robots move at a constant speed, s and can transport only one target at a time. After delivering a target to the collection zone, in the SPIRALCPFA and SPOKECPFA robots return to the point where the search pattern was interrupted. In the RANDCPFA, robots choose with some probability p_s to either employ site-fidelity, i.e.

return to the location of the last target discovered, or alternatively the robot chooses a random ballistic trajectory away from the collection zone. Site-fidelity is inspired by the behaviour of foraging ants [26]. Note that because the SPIRALCPFA and SPOKECPFA return to the point in their search pattern where they were interrupted by collecting a target, they implicitly implement site-fidelity.

The robots do not communicate with each other in the algorithms we study[§] (apart from initial assignment of search trajectories to each robot in the deterministic algorithms). Robots can remember only the last location where they found a target and the location of the collection zone.

Our formal model uses probabilistic, rather than adversarial, target distributions and therefore allows tighter results than those previously published. Table I summarises our notation. We do not consider environmental effects, robot failures, sensor or actuation error, collisions or congestion. We have considered these factors in our previous work using experiments with real robots and realistic simulations [3]. Collisions and congestion are considered in simulations in Section III.

1) The Omniscient Central-Place Foraging Algorithm:

For our formal analysis, we need an appropriate metric to measure relative success or failure. Our metric is the time for complete collection achieved by each algorithm compared to the time taken by an idealised omniscient algorithm. A perfect CPFA, for complete collection, simply has to know the location of each target *a priori*. The foraging time of this omniscient algorithm is simply the time for the N robots to travel directly to each target and return to the collection zone. Robots search in parallel, and a centralised scheduling algorithm can ensure that each target is assigned to only one robot. The difference between the CPFAs performance and the perfect algorithm is an effective measure of each algorithms' price of ignorance.

Each omniscient robot deploying this ideal algorithm will take exactly $2d/s$ units of time to collect a target, which is located at distance d from the collection zone. Since the expected distance is $d = 2R/3$ to a target, the total expected time for complete collection by this ideal algorithm is $4Rf/(3Ns)$.

2) *Price of Ignorance*: For a particular problem instance and a given foraging algorithm A , the price of ignorance metric, denoted $\chi(A)$, is defined as the time taken by A to collect all targets[¶] divided by that taken by the perfect algorithm:

$$\chi(A) = \frac{T_{\text{tot}}(A)}{4Rf/(3Ns)} = \frac{3NsT_{\text{tot}}(A)}{4Rf} \quad (1)$$

Thus, $\chi(A)$ must be at least 1 for any algorithm, and the most efficient algorithms are closest to 1. We summarise our

[§]As demonstrated by Rybski et al. [19], implementation of swarm communication in real systems may not show the expected improvement. Communication by real error-prone robots interpreting a noisy environment can harm foraging performance by misdirecting foraging resources, a phenomena termed *misinformation cost* by Pitonakova et al. [27].

[¶]We refer to this as the time to complete collection for algorithm A . Note that we do not address the problem of protocol termination in this paper and only measure the time until the last target is collected.

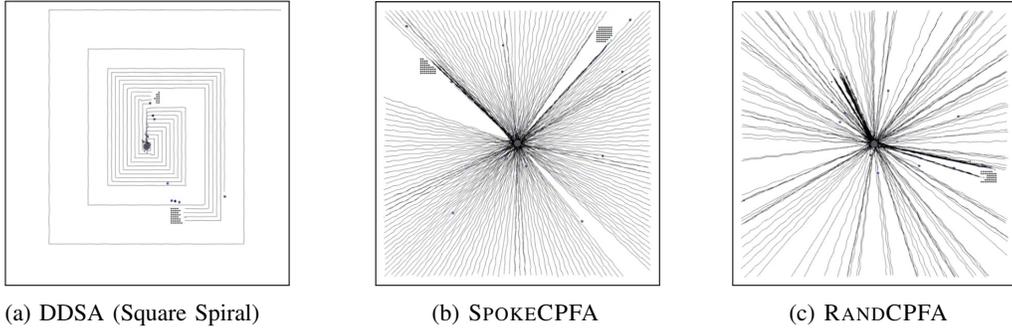


Fig. 1: The three foraging strategies in simulation. Targets were initially placed in $m = 2$ clusters with 64 targets each. For clarity only search paths are shown while transport paths are omitted.

main results for the price of ignorance of three CPFAs in Table II.

B. Distributed Archimedean Spiral Algorithm

The SPIRALCPFA (Figure 2) uses interleaving Archimedean spirals as robot search trajectories. Each robot starts at the centre, if it finds a target, it takes it back to the collection zone and then returns to the location where it found the target and resumes its search. If the robot hits an arena boundary, it completes the circuit at the arena boundary and then stops foraging. The spiral path is unique to each robot foraging in the arena and collectively guarantees full arena coverage. Even though we analyse a distributed Archimedean spiral, our results apply, within a constant factor, to the square spirals of the DDSA (Figure 1a) that we simulate in Section III.

The turning angles, θ , of robots following a distributed Archimedean-spiral are defined to be parametric in time, t such that $\theta = t + \frac{2\pi i}{N}$, resulting in the movement vector $\langle x, y \rangle = \frac{tNr}{\pi} \langle \cos \theta, \sin \theta \rangle$. In our proofs we use the polar representation.

Our main results are summarised in the theorem below.

Theorem 1. (SPIRALCPFA Analysis) *The following holds for foraging using SPIRALCPFA:*

- (1) *Using only a single robot, the expected price of ignorance of SPIRALCPFA is at most $1 + \frac{2.25R}{rf} - \frac{1}{2f}$.*
- (2) *Multiple robots reduce the expected price of ignorance of SPIRALCPFA to at most $\sqrt{2} + \frac{1.67R}{rf} - \frac{N}{2f}$. This bound is tight up to constant factors.*

Proof: [Proof Overview] The Archimedean Spiral is governed by the equation $\ell = \frac{r\theta}{\pi}$, where (ℓ, θ) is the polar co-ordinate pair of the spiral path traversed by the robot and r is the detection radius of the robot. Note that $2r$ is the separation between successive turns of the spiral, which is maintained in order to provide complete coverage of the arena.

1) *Proof Sketch for Theorem 1 (1):* Let $(\ell_{\max}, \theta_{\max})$ be the angular co-ordinates for the last point of the spiral that fits into the circular arena. Since the distance between $(0, 0)$ and $(\ell_{\max}, \theta_{\max})$ is R , we obtain $\theta_{\max} = \frac{R\pi}{r}$. Let L be the total length of the spiral path traversed by the robot. We can

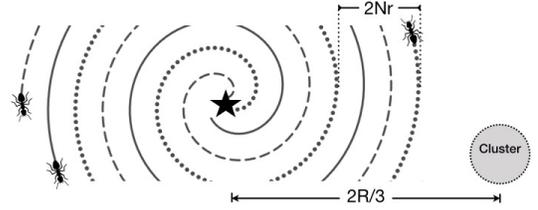


Fig. 2: Foraging path for $N = 3$ bots using SPIRALCPFA, with a cluster placed at the expected distance of $2R/3$ from the collection zone.

bound L as follows:

$$L \leq \frac{r}{\pi} \int_0^{\theta_{\max}} \sqrt{\theta^2 + 1} d\theta \leq \frac{r}{\sqrt{2}\pi} \theta_{\max}^2 < \frac{3R^2}{r} \quad (2)$$

Note that in SPIRALCPFA, every time the robot finds a target, it first carries that target back to the collection zone and then resumes its search from the point where it found the last target. Thus, to calculate the effective search time for a robot, we also consider the distance it needs to travel back to the cluster from the collection zone for each of the remaining $f - 1$ targets. This gives $E[T_s(\text{SPIRALCPFA})] = \frac{L}{s} + (f - 1) \frac{2R}{3s} \leq \frac{3R^2}{3s} + \frac{2R(f-1)}{3s}$. Note that the expected transport time is $\frac{2Rf}{3s}$, since it takes $\frac{2R}{3s}$ time in expectation to transport a single target back to the collection zone. Hence, the expected total completion time for SPIRALCPFA is at most $\frac{3R^2}{rs} + \frac{2R(f-1)}{3s} + \frac{2Rf}{3s}$. This gives the expected price of ignorance as stated in the theorem.

2) *Proof Sketch for Theorem 1 (2):* SPIRALCPFA can be easily adapted for multiple robots by ensuring that each robot travels along a separate Archimedean Spiral path in a way that no two spirals ever intersect. For example, in the steady state Figure 2 shows the foraging trajectory for $N = 3$ robots. Based on this, each of the robots will now traverse a path where the distance between successive spirals is $2rN$, governed by the equation $\ell_i = \frac{rN}{\pi} \theta_i + (i - 1) \frac{r}{\pi}$. Here, (ℓ_i, θ_i) are the polar co-ordinates of the spiral path along which the i^{th} robot travels in the arena for $i \in \{1, 2, \dots, N\}$.

Let $(\ell_{i, \max}, \theta_{i, \max})$ be the angular co-ordinates for the last point traversed by the i^{th} robot in the arena and L_i be the length of the Archimedean Spiral so traversed. Similar to the analysis for a single robot, we obtain the following $\theta_{i, \max} = \left(\frac{R\pi}{r} - (i - 1)\right) \frac{1}{N}$ and $L_i \leq \pi R^2 / (\sqrt{2}rN)$.

To compute the expected total transport time, note that each robot traverses an area of $2r$ per unit length of the path it travels during the search. Thus, the total area traversed by the i^{th} robot is $2rL_i \leq \sqrt{2\pi R^2}/N$. This corresponds to $\frac{\sqrt{2}}{N}$ fraction of the arena and hence, the expected number of targets collected by any robot is at most $\frac{\sqrt{2}f}{N}$. This way, we obtain $E[T_t(\text{SPIRALCPFA})] = \frac{2\sqrt{2}Rf}{3Ns}$.

Next, we bound the expected total search time. Similar to the discussion for the single robot, we can generalise the total expected search time for multiple robots as follows:

$$\begin{aligned} E[T_s(\text{SPIRALCPFA})] &= \max_i \left(\frac{L_i}{s} + E[T_t(\text{SPIRALCPFA})]_i - \frac{2R}{3s} \right) \\ &\leq \max_i \left(\frac{L_i}{s} \right) + E[T_t(\text{SPIRALCPFA})] - \frac{2R}{3s} \end{aligned} \quad (3)$$

Note that it holds that,

$$E[T_{\text{tot}}(\text{SPIRALCPFA})] = E[T_t(\text{SPIRALCPFA})] + E[T_s(\text{SPIRALCPFA})]$$

This gives,

$$\begin{aligned} E[T_{\text{tot}}(\text{SPIRALCPFA})] &\leq \max_i \frac{L_i}{s} + 2E[T_t(\text{SPIRALCPFA})] - \frac{2R}{3s} \\ &< \frac{\pi R^2}{\sqrt{2}rNs} + \frac{4\sqrt{2}Rf}{3Ns} - \frac{2R}{3s} \end{aligned}$$

The bound on the price of ignorance, as stated in the theorem, then immediately follows.

C. Spoke Central Place Foraging Algorithm

In the SPOKECPFA (Figure 1b), the searchers move radially away from the collection zone until they find a target or reach the arena boundary. When a searcher returns to the collection zone it increments its angle of departure slightly for the next spoke. The radial search progresses around the collection zone like the hands of a clock. This sweeping mechanism will cover the entire space in the arena. This reproduces the spoke strategy from the NASA Swarmathon; however, in our analysis robots return along a different path which we hypothesis will increase efficiency. The impact of this difference is discussed in Section IV.

Successive turns ensure that the maximum distance between any two spokes is at most twice the detection radius of the robots so that no cluster (even containing only one target) will be undetected. Thus, we set the sweep angle $\Theta = 4 \sin^{-1}(r/R) \geq 4r/R$ for a total of $2\pi/\Theta \leq \pi R/(2r)$ sweeps. Hence, the distance travelled by the robot to sweep the entire arena is $4R\pi/\Theta \leq \pi R^2/r$ (here, we assume that the robot moves along one spoke and returns along the next to further reduce the time by a factor of 2). Note that the search time also takes into consideration the time taken by the robot to travel back to the last location where it found the target[‡], which in expectation is $2Rf/(3s)$, resulting in a

[‡]Note that the time taken for the general case where multiple targets can exist along a single spoke is never more than the bound obtained here. This is because the distance covered by the robot that takes multiple trips along a single spoke (for, say, f' targets along the spoke) until it reaches the end of the arena is bounded from above by that covered by f' single trips along different spokes.

total search time of at most $\pi R^2/rs + 2Rf/(3s)$. The total transport time for f targets is $2Rf/(3s)$, in expectation. With multiple robots, the total time can be reduced by a factor of N by assigning each robot to its own sector. Since the clusters are uniformly distributed, each robot forages only $1/N$ fraction of the total number of targets, in expectation, and covers a search area that is $1/N$ fraction of the whole arena. Thus, expected price of ignorance of SPOKECPFA is $\frac{3\pi R}{4rf} + 1$.

D. Randomised Ballistic Central-Place Foraging Algorithm

In RANDCPFA (Figure 1c), the robot chooses a random direction from the collection zone and moves in a straight line path along that direction until either a target is found (either in direct line of sight or within the detection radius) or a pre-specified distance is reached (which in this case is the arena radius R). We state the main results for this algorithm in the theorem below and provide a proof sketch thereafter.

Theorem 2. (RANDCPFA Analysis) *The following holds for stochastic foraging using RANDCPFA:*

- (1) *Using only a single robot, the expected price of ignorance of RANDCPFA is at most $\frac{\pi R}{r} - \frac{1}{2}$ without site fidelity or considering the effect of target depletion.*
- (2) *Deploying site fidelity is expected to strictly decrease the price of ignorance to at most $\frac{m}{f} \left(\frac{\pi R}{r} \right) + \frac{1}{2} \left(1 - \frac{m}{f} \right)$, without considering the effect of target depletion.*
- (3) *The depletion of piles over time (due to foraging) causes the price of ignorance to increase to at most $\frac{(2\pi \ln 2)NR}{r} + \frac{1}{2}$, without site fidelity.*
- (4) *When both site fidelity and depletion effects are considered, the expected price of ignorance of RANDCPFA is at most $\frac{\pi R}{r} - \frac{N}{2f}$.*

Proof: We begin by computing the likelihood p of finding at least one target cluster in a random chosen direction from the collection zone (see Figure 3). It is trivial to see that $p = 1 - \left(1 - \frac{\theta}{2\pi} \right)^m$, where $\theta = 2 \sin^{-1} \left(\frac{3}{2R} \left(r + \sqrt{\frac{fa}{m\pi}} \right) \right)$. This is because the target cluster is detected as soon as its outer boundary falls within the detection radius of the robot [5]. The inverse sine expression can be simplified to obtain the following two bounds:

$$\theta \geq \frac{3r}{R} \quad (5)$$

$$\frac{3r}{2\pi R} \leq p \leq \frac{3mr}{\pi R} \quad (6)$$

1) *Proof Sketch for Theorem 2 (1):* If the target is not found, the robot travels for $2R/s$ units of time before reaching the collection zone again; otherwise, it travels for expected $2R/(3s)$ units of time before finding a target cluster. Assuming only a single robot in the system, recall that with probability p , the robot finds at least one target cluster. This causes the expected search time for the robot to be the following:

$$T_s = \frac{1}{s} \sum_{i=0}^{\infty} (1-p)^i p \left(2Ri + \frac{2R}{3} \right) = \frac{2R(3-2p)}{3ps} \quad (7)$$

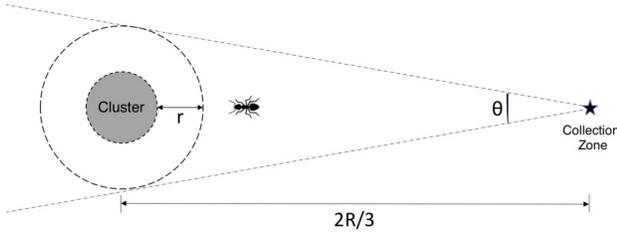


Fig. 3: Relationship between the angular region in which the cluster is detected in a single ballistic walk and the area of the cluster and detection radius of the robot.

Without site fidelity, then to find all f targets, the robot needs to make f such identical and independent trips, resulting in a total search time of fT_s .

Now, observe that once a target cluster is found, the robot moves in a straight line path to the collection zone, costing $\frac{2R}{3s}$ units of time. Thus, the total transport time taken by the robot to collect all targets in the arena is $T_t = \frac{2fR}{3s}$. Adding the total search time and the transportation time, we obtain the total foraging time for RANDCPFA to be $T_{\text{total}} = fT_s + T_t = \frac{2Rf(3-2p)}{3ps} + \frac{2fR}{3s} = \frac{2Rf(3-p)}{3ps}$. This allows us to bound the price of ignorance using Equation (1) as follows:

$$\chi(\text{RANDCPFA}) = \left(\frac{2Rf(3-p)}{3ps} \right) \left(\frac{3s}{4Rf} \right) = \frac{3-p}{2p} \quad (8)$$

Using the lower bound on p from Equation (6), we obtain the the result as stated.

We obtain a tighter bound using a coupon-collector argument in Section II-E. However, it becomes non-trivial to extend this argument for the case of site-fidelity and target depletion and hence, we continue the remaining proofs in this theorem using the same technique as discussed above.

2) *Proof Sketch for Theorem 2 (2)*: We say that the robot exhibits site fidelity $p_s \in [0, 1]$ if each time it finds a target at a particular site, it returns to that site for its next trip with probability p_s . Otherwise, it chooses a random direction to move. We then denote $\chi(p_s)$ to be the price of ignorance given site fidelity p_s .

Ignoring the effects of target depletion, the expected search time to find a target cluster for the first time is the same as before. However, for each of the remaining $\frac{f}{m} - 1$ targets in that pile, the robot takes in expectation $p_s(2R/(3s)) + (1-p_s)T_s$ units of time. This is because the robot returns to the same location with probability p_s and performs a Ballistic search otherwise. Hence, the expected search time to completely consume a target cluster with site fidelity is $T_s + (f/m - 1)(p_s(2R/(3s)) + (1-p_s)T_s)$, which can be rearranged to $(1 + (f/m - 1)(1 - p_s))T_s + p_s(f/m - 1)(2R/(3s))$. The transport time for these targets remains the same. Hence, the total expected time taken by the robot to collect all the targets in the arena is $(mp_s + f(1 - p_s))T_s + p_s(f - m)\frac{2R}{3s}$. Using Equations (6) and (7), we show that the expected price of ignorance is at most $\frac{m}{f} \left(\frac{\pi R}{r} \right) + \frac{1}{2} \left(1 - \frac{m}{f} \right)$, as stated in the theorem.

3) *Proof Sketch for Theorem 2 (3)*: For mathematical simplicity, we perform our analysis assuming only one target per pile ($f = m$) and no site fidelity (so that each robot

collects m/N piles in expectation). We begin with computing how the probability of finding target clusters degrades as they are depleted. Let p_n be the probability that in a single trip of the robot, at least one target cluster is encountered, assuming the arena currently has n unexplored target clusters. Then, $p_n \approx \frac{n\theta}{2\pi}$ (when θ is sufficiently small compared to $2\pi/m$). Given this value of p_n , the (expected) search time is at most $\frac{2R(1+2p_n)}{3s(1-p_n)}$ to find a single target cluster. Thus, to find the time to search for all the m/N target clusters, we use an integral approximation and assume $N < m$ to obtain the following (here T_s^{depl} denotes the total (expected) search time with depletion effects and $q_n = 1 - p_n$):

$$T_s^{\text{depl}} \leq \sum_{k=1}^{m/N} \left(\frac{2R(1+2p_{m-kN})}{3s(1-p_{m-kN})} \right) \leq \frac{8\pi m R^2 \ln 2}{3rs} \quad (9)$$

Using Equation (5), we obtain the following:

$$\chi \leq \frac{3Ns}{4Rm} \left(\frac{8\pi m R^2 \ln 2}{3rs} + \frac{2Rm}{3Ns} \right) \leq \frac{(2\pi \ln 2)NR}{r} + \frac{1}{2} \quad (10)$$

4) *Proof Sketch for Theorem 2 (4)*: Let p_f be the probability of discovering the target cluster in one Ballistic walk by the robot when the number of targets in the target cluster is f . We know that $p_f = \theta_f/2\pi$, where θ_f has the same form as in Equation (5). Let $T_{s,f}^{\text{NoSF}}$ be the expected time (without site fidelity) for a single robot to find the target cluster when it has f targets. Note that a single robot collects $1/N$ fraction of the total targets in expectation. Then, we obtain the following:

$$T_{s,f}^{\text{NoSF}} = \sum_{i=1}^{f/N} T_{s,i}^{\text{NoSF}} = \sum_{i=1}^{f/N} \frac{2R(3-2p_i)}{3sp_i} \leq \frac{4Rf}{3Ns} \left(\frac{\pi R}{r} - 1 \right) \quad (11)$$

where the last inequality follows from the bound in Equation (6). Adding the effect of site fidelity (where p_s is the probability that the robot will return to the same location that it last located a target), we denote by $T_{s,f}(p_s)$ the expected search time in this case. Then, we obtain the following:

$$\begin{aligned} T_{s,f}(p_s) &= T_{s,f}^{\text{NoSF}} + \sum_{i=1}^{(f/N)-1} \left(\frac{2Rp_s}{3s} + T_{s,i}^{\text{NoSF}}(1-p_s) \right) \\ &\leq \frac{4Rf}{3Ns} \left(\frac{\pi R}{r} - 1 \right) + \frac{2Rf}{3Ns} - \frac{2R}{3s} \end{aligned} \quad (12)$$

Note that the bound above holds for $p_s = 1$ (perfect site fidelity). Adding the expected transport time, this gives the upper bound on the total expected foraging time of RANDCPFA as $\frac{4Rf}{3Ns} \left(\frac{\pi R}{r} \right) - \frac{2R}{3s}$. This causes the price of ignorance to be at most $\frac{\pi R}{r} - \frac{N}{2f}$.

E. Tighter Analysis for RANDCPFA

We now discuss a method for obtaining a tighter bound on the expected foraging time of the RANDCPFA, ignoring the effects of target depletion and site fidelity. Observe that in the proof for Theorem 2, we assumed that the expected time to collect f targets is at most f times the expected time

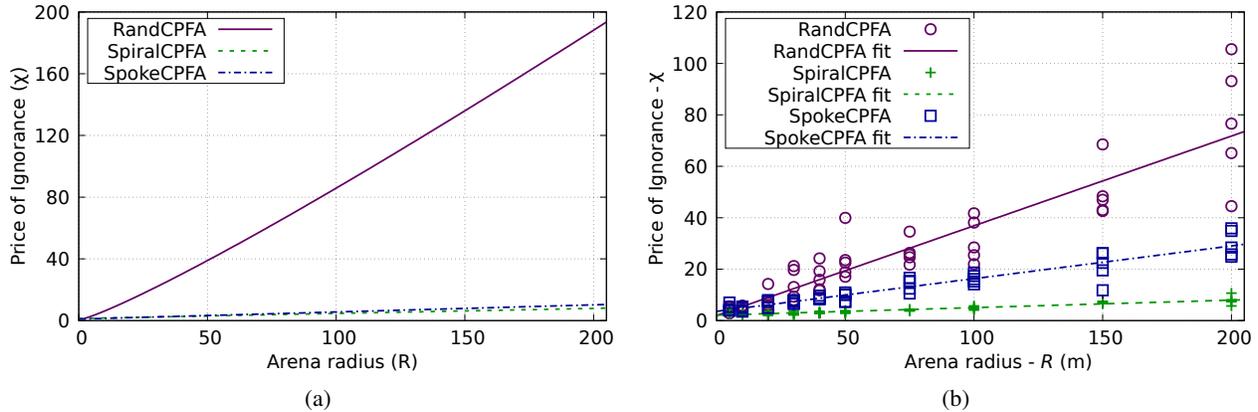


Fig. 4: Theory and simulation agree in the ordering of CPFA performance as a function of arena size ($N = 10$, $r = 0.2$, $f = 256$, and $m = 4$). (a) Comparison of theoretical predictions. (b) Simulations performed using the ARGoS simulator. For large arenas SPIRALCPFA performs much better than RANDCPFA and better than SPOKECPFA in simulation and in theory. For the linear fits $R^2 = 0.842$ for SPIRALCPFA, $R^2 = 0.897$ for SPOKECPFA and $R^2 = 0.841$ for the RANDCPFA. All arena dimensions are given in meters.

to collect a single target. In this simplification, the robots may make multiple passes over the entire arena to achieve complete collection.

However, observe that as soon as the entire arena is visited once, all targets would have been collected and no further foraging is required. This allows us to use the expected time to cover the entire arena as a tighter upper bound on the search time for RANDCPFA, compared to that obtained in Theorem 2 (1). To obtain this bound, we bound the number of distinct ballistic trajectories by $\frac{2\pi}{\theta}$, where θ is the same as that in Equation (5). Since each trajectory can be traversed multiple times independently and identically, the expected number of times each trajectory is traversed at least once is at most $\frac{4\pi}{\theta} \log\left(\frac{2\pi}{\theta}\right)$, which is obtained similar to the analysis of coupon collector problem [28]. The maximum time it takes to complete each trajectory is $\frac{2R}{s}$, and hence, a tighter upper bound on the total foraging time is now at most $\left(\frac{2R}{s}\right) \left(\frac{4\pi}{\theta}\right) \log\left(\frac{2\pi}{\theta}\right) + \frac{2Rf}{3s} \leq \frac{8\pi R^2}{3sr} \log\left(\frac{2\pi R}{3r}\right) + \frac{2Rf}{3s}$. This allows us to bound the price of ignorance by $\frac{2\pi R}{rf} \log\left(\frac{2\pi R}{3r}\right) + \frac{1}{2}$, which is significantly lower than in Theorem 2 (1) (see Figure 5a for an empirical comparison).

III. EXPERIMENTAL METHODS

To validate the formal analysis, we ran experiments using the ARGoS simulator for a variant of each algorithm implemented for a square arena with $N = 10$ robots. The “foot-bot” robot, programmed to move like the iAnt robot from [15], was used for all experiments. The experiments measured the time to complete collection in an environment with $f = 256$ targets arranged in $m = 4$ clusters, of 64 items each, placed uniformly at random in the arena. Five replicates were run for each arena size. Robots were able to detect and pick up a target if they came within $r = 0.2$ m of it. For the RANDCPFA experiments, the robot always use site fidelity ($p_s = 1$). Results from these experiments are shown in Section IV Figure 4b.

IV. SIMULATION RESULTS

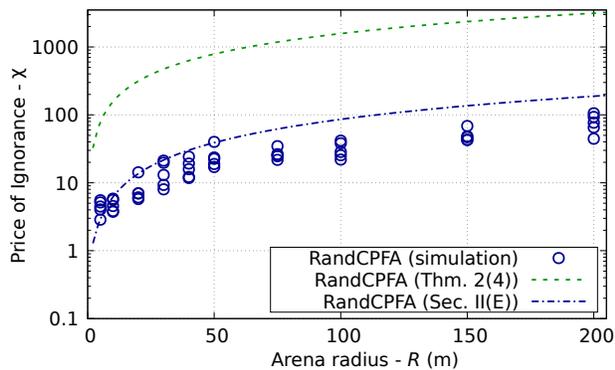
Figure 4a shows the relative price of ignorance for each algorithm given in Section II. As predicted, our experiments show that SPIRALCPFA has the lowest cost of ignorance for large arenas with greater cost of ignorance for SPOKECPFA and greatest for RANDCPFA. We also observe that the variance in price of ignorance is very low for SPIRALCPFA in simulation whereas for SPOKECPFA and RANDCPFA the price of ignorance is highly variable, demonstrating that the performance of these algorithms is sensitive to the placement of clusters.

Figure 5a shows that performance of RANDCPFA in simulation closely matches the tighter bound given in section II-E. Figure 5b shows that the simulation results for SPIRALCPFA match the formal analysis well. SPOKECPFA, however, performs considerably worse than the theoretical predictions.

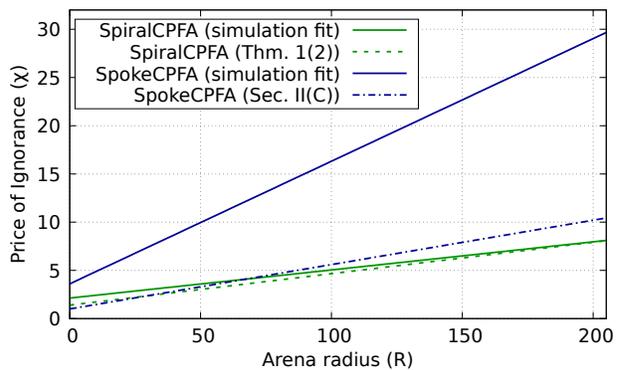
Two factors help to explain why simulation performs worse than the theoretical bound. In the analysis, robots search different spokes on the outbound and return trips, but in our implementation, each robot’s return spoke is the same as the outbound one, increasing search time by up to a factor of 2. Additionally, while the theory assumes that transport of targets is equally divided among the robots, our simulations used a relatively small number of robots and clusters. Since the number of clusters was smaller than the number of robots, the robots could not fully parallelize the search.

V. DISCUSSION AND CONCLUSION

Our theoretical analysis and experiments in simulation help to explain the effectiveness of spiral algorithms observed in previous simulations and experiments with physical robots [2], [3]. Our simulation results are consistent with those predicted by our formal analysis. We show that SPIRALCPFA is expected to outperform both SPOKECPFA and RANDCPFA. We attribute this to the non-redundancy in the search path of SPIRALCPFA, where the robot continuously



(a)



(b)

Fig. 5: Comparison of observed price of ignorance to the upper bounds derived in Section II. (a) Theory vs. simulation for RANDCPFA showing the improvement given by the analysis in Section II-E. (b) Combination of Figure 4a and Figure 4b focusing on the agreement between theory and simulation in demonstrating that the SPIRALCPFA performs better than the SPOKECPFA for larger arenas and matches the theoretical predictions very well. All arena dimensions are given in meters.

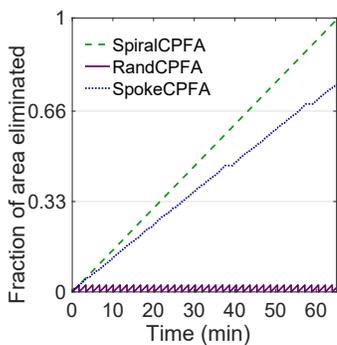


Fig. 6: Fraction of area explored by the robot over time. SPIRALCPFA never re-searches the same area, which reduces the price of ignorance.

eliminates area to search for targets with every step and no two robots have overlapping search areas. This is not the case for SPOKECPFA which repeatedly visits points near the centre of the arena (Figure 1b). Overlap in search is even worse in RANDCPFA since each successive ballistic walk is made independently of previously foraged areas. We plot an approximation of the fraction of area eliminated for search over time for the three algorithms in Figure 6. We observe that RANDCPFA ignores all previously obtained information and performs each ballistic walk as if it were foraging in a completely unknown arena. On the other hand, SPIRALCPFA shows a monotonic increase in the information gained about what parts of the arena do not contain any more targets and chooses never to traverse those areas again. SPOKECPFA increases more slowly over time due to overlap near the centre.

The theoretical analysis reveals why target depletion and site fidelity are important to consider in RANDCPFA. As targets are collected, clusters become smaller and it becomes more challenging for the robots to find them. To the best of our knowledge, we make the first attempt to model the effect of depletion in collaborative foraging and prove that

TABLE II: Summary of various analytical results in this paper. See Table I for notation. Ranked according to the least upper-bound on expected performance (lower is better).

Algorithm	Rank	Collection Time	Price of Ignorance
Ideal	1	$\frac{4Rf}{3Ns}$	1
SPIRALCPFA	2	$\frac{\pi R^2}{\sqrt{2}Nrs} - \frac{2R}{3s} + \frac{4\sqrt{2}Rf}{3Ns}$	$\frac{3\pi R}{4\sqrt{2}rf} - \frac{N}{2f} + \sqrt{2}$
SPOKECPFA	3	$\frac{\pi R^2}{Nrs} + \frac{4Rf}{3Ns}$	$\frac{3\pi R}{4rf} + 1$
RANDCPFA	4	$\frac{4\pi R^2 f}{3Nrs} - \frac{2R}{3s}$	$\frac{\pi R}{r} - \frac{N}{2f}$

when target depletion is taken into account, one can expect the price of ignorance to increase with N . In RANDCPFA a larger number of robots on one hand collect targets faster (by working in parallel), but they also deplete the arena at a rate which significantly slows down foraging as time progresses. The counter-intuitive result that the price of ignorance increases with N is because the omniscient algorithm pays no cost for search, while RANDCPFA pays an additional cost as clusters shrink and therefore become harder to find. Site fidelity compensates for target depletion because robots only have to find each cluster once. This is evident in the analysis because when both site fidelity and depletion are accounted for, the price of ignorance no longer increases with N (compare Theorem 2 (3) with (4)).

Thus, our analysis shows that site fidelity strongly counteracts the effect of target depletion which greatly improves performance of RANDCPFA. Empirically, target depletion makes finding targets exponentially more difficult over-time [29]. Site fidelity has been observed to improve foraging performance in stochastic algorithms in simulation and in biological field studies [15], [20], [26], [30]. Our formal analysis explains this phenomenon and suggests that site fidelity may be an important component of stochastic foraging algorithms more generally.

The effect of target depletion is further exacerbated in simulation because robots may bore a ‘tunnel’ through

the clusters, resulting in two smaller clusters that must be rediscovered before all resources can be collected, an effect not explicitly captured by our formal model. We note that the deterministic algorithms considered in this paper are resistant to this effect of target depletion by design (for example Figure 1a). This is because the search trajectories cover the entire arena in a fixed time and do not revisit previously foraged areas while searching for more targets. Thus, when targets are depleted, they have no effect on the time taken by ongoing search for other targets.

The theoretical results presented in Table II allow us to not only quantify the efficiency of each algorithm, but also help us to understand the relative impact of search cost and transport cost. We provide insight into how site fidelity, a common strategy among social insects [20], mitigates the effect of target depletion on foraging time. Our analysis helps quantify the importance of keeping oversampling in search to a minimum, which is an argument in favour of deterministic search for limited-memory systems.

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